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Application of Fixed-Point Algorithm in Parallel Systems

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Abstract: Fixed point algorithm is a powerful method to determine more accurate solutions to dynamical systems and widely used in analysis, algebra, geometry, and logic which is available all over the world anywhere and anytime. Convergence of fixed point iteration plays vital role in the solution of problems. This paper introduces about fixed point algorithm and fixed point iteration with it applications. We studied some fixed point iteration methods which can be used in parallel systems. We assumed that each problem of parallel systems can be expressed or solved using the fixed point algorithm. For generating the parallel grid of processor four iterative algorithms or methods can be used.

Keywords: Fixed-Point, Fixed-Point Algorithm, Fixed-Point Iteration, Chain Point, Attractive Fixed-point.

1. INTRODUCTION

Discussion on the computation of fixed-points or fixed-point algorithms occurs in just every field of computer science (i.e. geometry, Algebra, Analysis, and algorithmic logic). The widespread use of its algorithms is due to the fact that the semantics of recursion can be described by fixed points of functions. Of course, the treatment of fixed points in mathematics goes well back before their first use in computer science as: Fixed-Point occur in analysis, algebra, geometry, and logic. One of the first occurrence of fixed points in the field of the theory of automata and were probably the equation characterization of regular and context-free languages at least solution to right-linear and polynomial fixed point equations [2][3] . The theorem about existence and properties of fixed points is also known as fixed point theorem [3]. Many results in the theory of automata and languages can be derived from basic properties of fixed points [2].

Definition-1: Suppose X is a set and f is a function which maps X to X. A fixed point of f is a point $x \in X$ such that $f(x) = x$.

I.e. If f is defined on the real number by $\mathbf{f(x) = x^2 - 7x + 10}$ then on solving their e.g. we know that x=2 and x=5 are root of the equation.

Let us consider $\mathbf{f(x) = x}$.

Where $\mathbf{x = \frac{x^2+10}{7}}$

Then $x = 2$ and $x = 5$ are two fixed point of $f(x)$

Definition-2: A fixed point of a function is an element of a function's domain that is mapped to itself by the function[7].

Definition-3: A fixed point for a function $f(x)$ is a value x_0 in the domain of the function such that $f(x_0) = x_0$. We say that the function $f(x)$ fixes the value x_0 [6].

Definition-4: Let C be a nonempty bounded closed convex subset of Banach space X. A mapping $T: C \rightarrow C$ is said to be nonexpansive if

$$||Tx - Ty|| \leq ||x - y|| \quad (1)$$

For all x, y in C. It has been shown that if X is uniformly convex, and then every non-expansive mapping $T: C \rightarrow C$ has a fixed point [5].

Definition-5: If a procedure $\emptyset(x)$ crossed by $y = x$ then, the x co-ordinate of a crossing is known as fixed point. This is shown below in figure 1.1. Where two circles one in another presents fixed point.

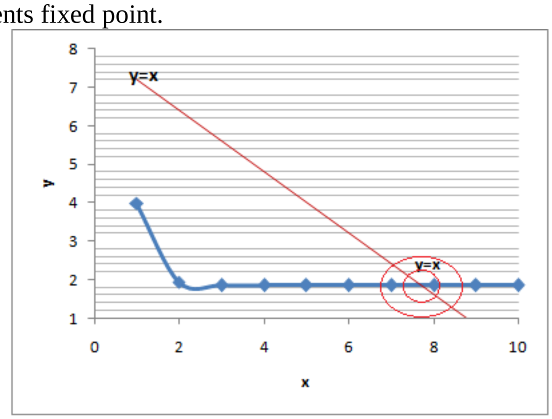


Fig:-1.1. A graphical example of fixed point

1.1. Geometrical representation

Geometrically the fixed point occurs where the graph of $y = f(x)$ crosses the graph of $y = x$. A function may be zero, one or more fixed points.

1.2. Analysis

In analysis fixed point play vital role for reckoning things. Such as programming languages compilers use fixed point calculation for program statement analysis, for example in data-flow analysis, which is often required for code optimization. In numerical analysis, by using fixed point iteration you can calculate fixed points of iterated functions. Suppose given a function a f defined on real numbers with real values and given a point x_0 in the domain of f, the fixed point iteration is $\boldsymbol{x_{n+1} = f(x_n), n = 0, 1, 2, \dots}$

To generate to the sequence $\boldsymbol{x_0, x_1, x_2, \dots}$ which is hoped to converge in a point *x*. if *f* is continuous, then one can prove that they obtained *x* is fixed point of f, i.e., $f(x) = x$.

1.3. Logic

Logician saul kripke makes use of fixed point in his influential theory of truth. He represents how one can generate a partially truth predicate (one that remain undefined for problematic sentence like “The sentence is not true”) by recursively defining “truth starting from the segment of a language that contains no occurrence of the word, and continuing until the process ceases to yield any newly well-defined sentences. (This takes a denumerable infinity of steps.) That is, for a language L, let L-prime is the language generated by adding to L, for each sentence S in L, the sentence "S is true." A fixed point is reached when L-prime is L; at this point sentences like "This sentence is not true" remain undefined, so, according to kripke, the theory is suitable for a natural language that contains its own truth predicate.

1.4. Algebra

In terms of algebra the fixed point(s) is (are) the solutions to the equation $f(x) = x$, [1].
For example the graphical solution of equation $y^2 = 2 + x \,\&\, y = x$ is discussed below. Here we show that the fixed point for above function is 2.

For equation $y2 = 2 + x$ the set of domain $is\ x = \{imagine, -2, -1, 2, 7, 14, 23\ \dots\dots\}\&\ set\ of\ range\ y =$

$$(imagine, 0, 1, 2, 3, 4, 5 \dots\dots\}$$

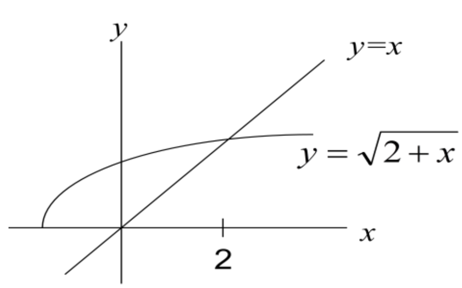


Fig:-1.2. A graph of algebraic fixed point

1.5. Attractive fixed point

An attractive fixed point of a function f is a fixed point x_0 of f such that for any value of x in the domain that is close enough to x_0, the iterated function sequence

$$x, f(x), f(f(x)), f(f(f(x))), \dots \qquad (2)$$

Converges to x_0. An expression of prerequisites and proof of the existence of such solution is given by the Banach fixed-point theorem [8].

1.6. Fixed point algorithm

The Fixed Point Algorithm (FPA) is an algorithm that generates a recursively defined sequence that will find the fixed point for a function under the correct conditions. One of the big advantages of the algorithm is that it is no very difficult to implement [6]. The Fixed Point Algorithm (FPA) uses a value x_0 (ideally chosen close to the fixed point you want to find) and a function f(x) and generate a recursively defined sequence given by:

$$\mathrm{x_n \text{ for } n = 0 \qquad and \qquad x_{n+1} = f(x_n) \text{for } n > 0.}$$

The FPA will be able to estimate a fixed point if and only if the sequence $\mathrm{x_n}$ converges [4].

There are several conditions that will that would imply convergence.

1. f(x) is increasing and bounded
2. f(x) is decreasing and contractive

To find the fixed point of g in an interval [a, b], given the equation $\mathrm{x = g(x)}$ with an initial guess $\mathrm{p_0 \in [a, b]}$:

1. n = 1
2. $\mathrm{P_n = g\,(P_{n-1})}$
3. If $|\,\mathrm{P_n - P_{n-1}}\,| < \in\ then$ 5
4. $\mathrm{n \to n + 1}$ go to 2
5. End of Procedure.

1.7. Fixed Point Iteration

In Numerical analysis, it is a method of computing fixed point by doing number of iteration to the function. Fixed point iteration according to his name and works may call as chain point, because the recent result of current solution works as an input of next solution. So it creates chain point in accurate values of solution. You can understand it by seeing below in numerical example.

1.7.1. Explanation of Fixed-Point Iteration

- Suppose g(x) is a function
- Now that we have established a condition for which g(x) has unique fixed-point in I (iteration), these remains the problem of how to find it. The Technique employed is known as fixed-point Iteration.
- To approximate the fixed-point of a function g, we choose an initial approximation $\mathrm{P_0}$ and generate the sequence $\{\mathrm{P_n}\}_{\mathrm{n=0}}^{\infty}$ by letting $\mathrm{P_n = g(P_{n-1})}$, For each n>=1
- If the sequence converges to P and g is continuous, then

- $P = \lim_{n\to\infty} P_n = \lim_{n\to\infty} g(P_{n-1}) = g(\lim_{n\to\infty} P_{n-1}) = g(p)$ and a solution to $x = g(x)$ is obtained.
- This Technique is called fixed-Point or functional iteration.

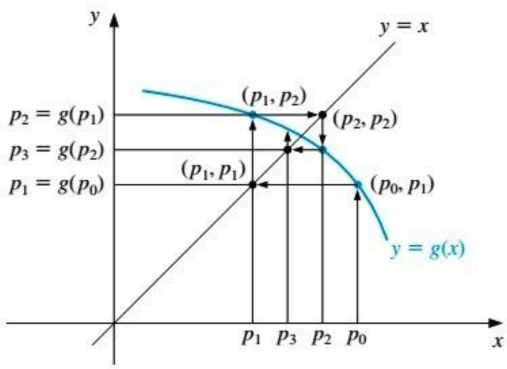


Fig:-1.3. Example Fixed-point iteration

1.7.2. Fixed Point Iteration Method [9]

The transcendental equation **f(x) = 0** can be converted algebraically into the form **x = g(x)** and then using the iterative scheme with the recursive relation

$$x_{i+1} = g(x_i), \quad i = 0, 1, 2, \ldots, \quad (3)$$

With some initial guess $\mathbf{x_0}$ is called the fixed point iterative scheme.

Algorithm – Fixed Point Iteration Scheme:

Given an equation $f(x) = 0$

Convert an equation $f(x) = 0$ into the form $x = g(x)$

Let the initial guess be x_0

Do

- C1. Fixing apriori the total number of iterations **N**.
- C2. By testing the condition $|x_{i+1} = g(x_i)|$ (where **i** is the iteration number) less than some tolerance limit, say epsilon, fixed apriori.

Numerical Example:

Consider the function $g1(x) = 10 / (3x - 1)$ and the fixed point iterative scheme

$$x_i + 1 = 10 / (3x_i - 1), i = 0, 1, 2 \ldots$$

Let see the initial guess x_0 be 1.3

i	0	1	2	3	4	5	6	…	10
x_i	1.3	3.84615 4	0.94890 5	5.41501 9	0.65595 0	10.3321 73	0.333372	..	0.32254 4

Table-1: Iterative process for function g1

So the iterative process with **g1** gone into an infinite loop without converging.

Consider another function **g2(x) = (x + 10)$^{1/4}$** and the fixed point iterative scheme

$$x_i + 1 = (x_i + 10)1/4, \quad i = 0, 1, 2 \ldots \quad (4)$$

Let the initial guess $\mathbf{x_0}$ be **1, 2, 4 and 5**

i	0	1	2	3	4	5	6
x_i	1	1.821 16	1.854 236	1.855 532	1.855 582	1.855 584	**1.855 585**
x_i	2	1.861 21	1.855 805	1.855 593	**1.855 585**		
x_i	4	1.934 336	1.858 658	1.855 705	1.855 589	**1.855 585**	
x_i	5	1.967 989	1.589 967	1.855 755	1.855 591	1.855 584	**1.8555 84**

Table-2: Iterative process function for g2

That is for **g2** the iterative process is converging to **1.855585** and **1.855584** with any initial guess.

Consider third function **g3(x) =(x+10)$^{1/2}$/x** and the fixed point iterative scheme

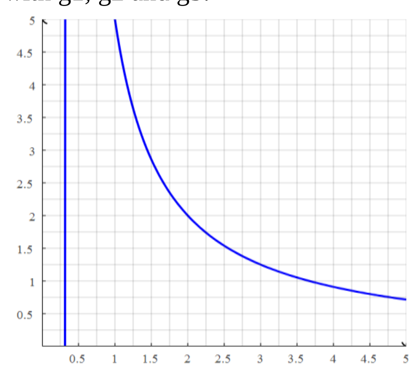
$$x_i + 1 = (x_i + 10)1/2 / x_i, \quad i = 0, 1, 2, \ldots \quad (5)$$

let the initial guess $\mathbf{x_0}$ be **1.8,**

i	0	1	2	3	4	5	6	7
x_i	1.8	1.98 4929	1.86 0625	1.85 5781	1.85 5592	1.85 5584	1.855 584	**1.85 5584**

Table-3: Iterative process for function g3

That is for g3 with any initial guess the iterative process is converging but very slowly to Geometric interpretation of convergence with g1, g2 and g3.



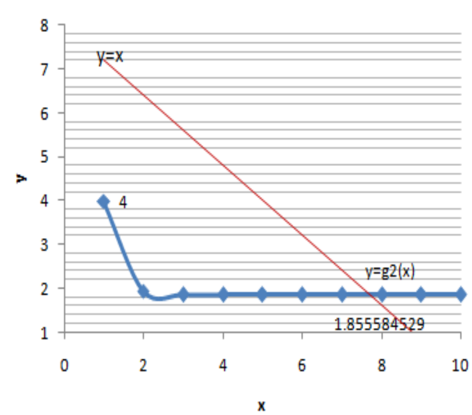


Fig: 1.5 (g2)

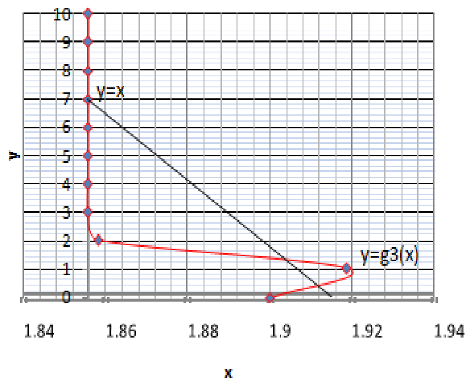


Fig: 1.6 (g3)

The graphs Figures 1.4(g1), 1.5(g2) and 1.6(g3) explain about the Fixed point Iterative Scheme with g1, g2 and g3 respectively for some starting closers. It's clear from the

- Fig 1.4(g1), the iterative process do not converges for any starting estimation.
- Fig 1.5(g2), the iterative process converges very quickly to the root which is the intersection point of $y = g2(x)$ as shown in the figure.
- Fig 1.6(g3), the iterative process converges but very slowly.

2. APPLICATION OF FIXED POINT ALGORITHM

Fixed point algorithm or fixed point theorem are many applications which play important role in computer science. Some applications are such as in-

2.1. Integral equations-This type of equation uses in engineering, mathematics and mathematical physics. Integral equation uses as a representation of formulas in the solution of differential equations. Some equations are as follows-

i. $g(x) = \int_a^b k(x,y)g(y)dy$

ii. $g(x) = f(x) + \int_a^b k(x,y)g(y)dy,$

iii. $g(x) = \int_a^b k(x,y)g(y)^2\, dy$

Above 'g' is unknown function and all other functions are known are called integral equations.

Normally linear integral equation in $y(x)$ can be written as

$$h(x)\, y(x) = f(x) + \int_a^{b(x)} k(x,t)y(t)\, dt \qquad (6)$$

2.2. Successive approximation-This is uses to determinative solution of integral differential and algebraic equations. In this case where $f\colon X \to X$ is a mapping then the sequence of successive approximation $x_{n=fx_n, n=0,1,2,\dots}$ converges to a fixed point of 'f'.

2.3. The vector of page rank values of all web pages is the fixed point of a liner transformation derived from the World Wide Web link structure.

2.4. The concept of fixed point can be used to define the convergence of a function.

2.5. A Very common and fundamental application of fixed point in computer science is to recursion theory and mathematically model loops.

3. PARALLELIZATION OF FIXED POINT ITERATION

The fixed point iteration algorithm can be used in parallel system for generating parallel grid of processors to solve the large and heavy processes. To generate the processors parallel grid mainly four iterative algorithm or methods can be used these are described ahead.

3.1. Jacobi's Iteration

In numerical linear algebra, the Jacobi method (or Jacobi iterative method) is an algorithm for determining the solution of a diagonally dominant system of linear equations. Each diagonal element is solved for, and an approximate value is plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization. The method is named after Carl Gustav Jacob Jacobi.

3.2. Gauss-Seidel Iteration

Gauss Seidel Iteration method utilize the latest iterative values if available and scan the mesh points symmetrically from left rows to right rows along with successive rows. You can understand with this conceptual figure and formulas.

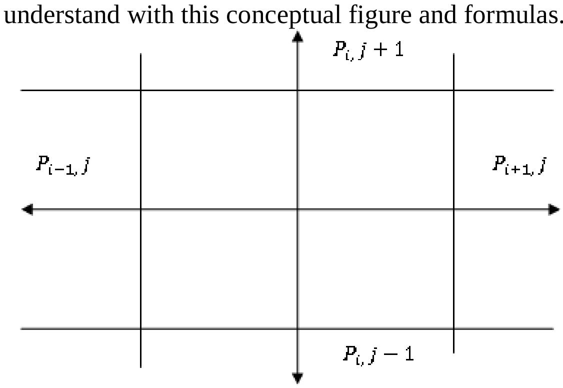


Fig: 1.7. Gauss-Seidel method

$$Pi^{(n+1)}, j = \frac{1}{4}\left[Pi - 1^{(n+1),} j + Pi + 1^{(n)}, j + Pi^{(n+1)}, j - 1 + Pi^{(n)}, j + 1\right] \qquad (7)$$

3.3. Standard Five Point Formula

$$P_{i,j} = \frac{1}{4}\left[p_{i+1,j} + P_{i-1,j} + u_{i,j+1} + P_{i,j-1}\right] \qquad (8)$$

Equation (8) based on figure 1.7 which shows that the value of $[P_{i,j}]$ is the average of its values at four neighbouring points East, West, North, and South. The formula (8) is known as standard five point's formula. This formula is also called Leibman's averaging procedure.

3.4. Diagonal Five Point Formula

If you want to good approximation for starting values at the mesh points in the iteration function the diagonal method could be used on the basis of below figure and their given formula

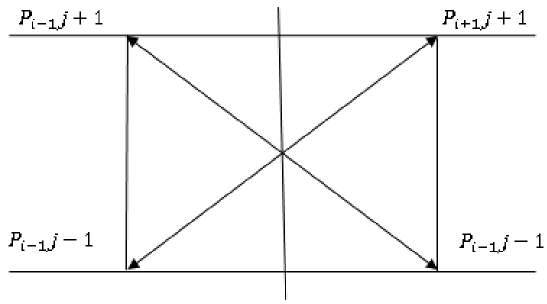


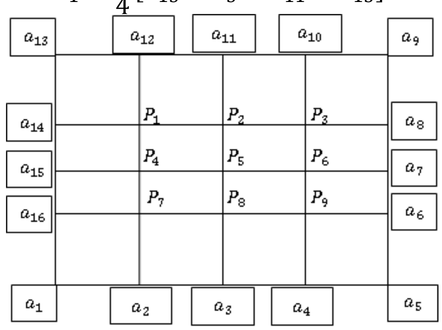
Fig: 1.8. Diagonal five point formula

$P_{i,j} = \frac{1}{4}[P_{i+1,j+1} + P_{i+1,j-1} + P_{i-1,j+1} + P_{i-}1, j-1]$ (9)

Process: Limit values are$a_1, a_2, a_3 \dots a_{16}$. The value P_5 at the centre and the values $P_{1,}P_3, P_{7,}P_9$ are computed using diagonal function so,

$$P_5 = \frac{1}{4}[a_1 + a_5 + a_9 + a_{13}]$$

$$P_1 = \frac{1}{4}[a_{15} + P_5 + a_{11} + a_{13}]$$



$$P_3 = \frac{1}{4}[P_5 + a_7 + a_9 + a_{11}]$$

$$P_7 = \frac{1}{4}[a_1 + a_3 + P_5 + a_{15}]$$

$$P_9 = \frac{1}{4}[a_3 + a_5 + a_7 + P_5$$

P_2, P_4, P_6, P_8 Are computed by standard five point method-

$$P_2 = \frac{1}{4}[P_1 + P_3 + P_5 + a_{11}],$$

$$P_4 = \frac{1}{4}[P_1 + P_5 + P_7 + a_{15}]$$

$$P_6 = \frac{1}{4}[P_3 + P_5 + P_9 + a_7]$$

$$P_8 = \frac{1}{4}[P_5 + P_7 + a_3 + P_9]$$

After using these iterative methods parallel grid of processors may be possible by which accurate solution of problem can be finding. Many other algorithms are used to the solution of parallel system of mathematical expression, optimization and some other problem which have the structure

$X\,(t+1) = f(x\,(t)), t = 0, 1 \dots,$ (10)

Where apiece $x(t)$an n-dimensional vector and f is is some function. The notation that is $x = f(x)$ where if the sequence $\{x(t)\}$ presented by the mathematical expression or above iteration converges to a limit $x*$, and if the function f is continuous, then $x*$ is a fixed point of f, if it satisfies $x* = f(x*)$.

The above iterative equation $X\,(t+1) = f(x\,(t))$ can be expand as

$x_i(t+1) = f_i(x_i(t), \dots., x_n(t)), \quad i = 1, \dots, n.$ (11)

The iterative algorithm $x = f(x)$ could be parallelized by letting apiece one of n processor update a many part of x according to above equation (10). In each iteration the i^{th} processor find the value of all parts of $x(t)$ on which f_i depends, computes the new value $x_i(t+1)$ and communicate to other processors in order to run next iteration.

Other way the iteration $x(t+1) = f(x(t))$ can be as

$x_j(t+1) = f_j(x(t)), \qquad j = 1, \dots., p,$ (12)

Where apiece f_j is a vector function which map R^n into R^{nj} and assign each one of p processor to update a different block-component according to equation (12) resulting parallel algorithm is called block-parallelized. Block-parallelization takes less communication requirement of an algorithm [10].

4. CONCLUSION

Fixed point algorithm is a type of tools which is available everywhere. In computer science and mathematics it frequently uses in analysis, algebra, geometry, and logic. Nature is also a big example of iteration which is fixed according to its time, angle and condition. If iteration convergence is any particular point then it is called fixed point. Convergence of any fixed point is also useful characteristic. Using fixed point with better convergence rate you can find good solution of any problem. Problem may be linear or in parallel way.

REFERENCES

[1] Esik. Z., (1980), "Identities in iterative algebraic theories Computational Linguistics and Computer Languages", 14:183–207.

[2] Esik Zoltan. (2009),"Fixed Point theory" Springer-Verlag Berlin, chapter 2, Page 29-65.

[3] Asati Alok, Singh Amardeep and Parihar C.L. (2013), "127 Years of Fixed point theory-A Brief Survey of development of fixed point theory" Vol 04.

[4] Fiacco A.V., (1974), "Convergence properties of local solutions of sequences of mathematical programming problems in general spaces", Journal of Optimization Theory and Applications Vol 13, 1–12.

[5] Kok-Keong Tan & Hong-Kun Xu, (1993), "Approximating fixed point of nonexpansive Mappings", by the Ishikawa iteration process, journal of Mathematical analysis and applications 178, 301-308.

[6] Burden R L & Faires J D, (2011), "Numerical Analysis" Dublin City University.

[7] Coxeter, H. S. M. (1942), "Non-Euclidean Geometry" University of Toronto Press. p. 36.

[8] Weisstein, Eric W. "Dottie Number", Wolfram MathWorld, Wolfram Research, Inc. Retrieved 23 July 2016.

[9] https://mat.iitm.ac.in/home/sryedida/public_html/caimna/transcendental/iteration%20methods/fixed-point/iteration.html.

[10] Ortega, J.M., and R.G. Voigt. 1985, "Solution of partial differential equations on vector and parallel computers", SiAM Rev.27:149-240.

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