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Coloring of Polyhedral Regular and Irregular Fuzzy Graphs

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Abstract— In this article we propose coloring of polyhedral regular and irregular fuzzy graphs.Also we have investigate some new concepts of polyhedral fuzzy graphs,Planar fuzzy graphs and coloring of polyhedral regular and irregular fuzzy graphs.We anayze some basic theorems related to these concepts.

Keywords— Fuzzy graphs, Coloring of fuzzy graphs,Polyhedral fuzzy graphs,Planar fuzzy graphs, Polyhedral regular fuzzy graphs,Polyhedral irregular fuzzy graphs**.**

I. INTRODUCTION

The origin of graph theory started with the konigsberg bridge problem in 1735.This problem led to the concept of the Eurelian graph. Euler studied the konigsberg bridge problem and constructed a structure that solves the problem that is referred to as an Aurelian graph. Currently concept of graph theory are highly utilized by computer science applications, especially in areas of computer science research, including data mining, image segmentation, clustering and networking.

Graph coloring is one of the most important problems of combinatorial optimization. The first basic definitions of regular fuzzy graph was proposed by M.Akram and W.Dudek[1].In 1987,Bhattacharya[2]introduced the concept of some remarks on fuzzy graphs.

Also K.Kalaiarasi and L.Mahalakshmi[3,4,5] defined basic definition and regular and irregular m-polar fuzzy graphs also introduced the basic concepts of coloring of regular and strong arcs fuzzy graphs.Finally sinisa T.Vrecica[7] introduced on polygons and polyhedral. In this paper we introduced coloring of regular and irregular polyhedral fuzzy graphs.

In Section I Contains the basic definitions and Section II Contains our proposal of the Definition Coloring of polyhedral regular and irregular fuzzy graphs.In Section III contains the established theorems.Section IV is our research of fuzzification to the coloring of regular and irregular planar fuzzy graphs and we examine many results.

II. Preliminaries

Definition 2.1.Fuzzy graph

A fuzzy graph is an ordered triple $G(V,\sigma,\mu)$ where V is a set of vertices $\{u_1,u_2,...u_n\}$ and σ is a fuzzy subset of V (i.e) $\sigma : V \to [0,1]$ and is denoted by $\sigma = \{(u_1,\sigma(u_1)),(u_2,\sigma(u_2)),...(u_n,\sigma(u_n))\}$ and μ is a fuzzy relation on σ (ie) $\mu(u,v) \le \sigma(u)\sigma(v)$.

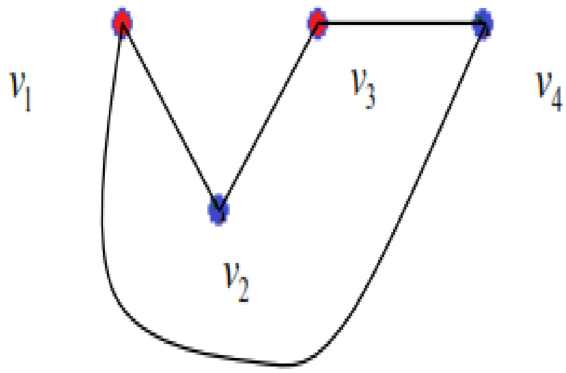
Definition 2.2. Coloring of fuzzy graphs:

A Coloring of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring).The set of all vertices with any one color is independent and is called a color class. A family $\Gamma\{\gamma_1,\gamma_2,.....,\gamma_k\}$ of fuzzy sets on a set V is called a $k-$ fuzzy coloring of $G=(V,\sigma,\mu)$ if (i) $\vee\Gamma=\sigma$ (ii) $\gamma_i \wedge \gamma_j = 0$ (iii) For every strong edge $(x,y)[(i.e)\mu(x,y)>0]$ of $G.\min\{\gamma_i(x),\gamma_i(y)\}=0 (1\le i\le k)$.

Definition 2.3. Coloring of Planar fuzzy graphs:

A fuzzy graph $G(V,\sigma,\mu)$ is said to be planar fuzzy graph if it can be drawn on a plane or sphere so that no two edges cross each other at a non-vertex point. A Coloring of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring).

Example 2.3.



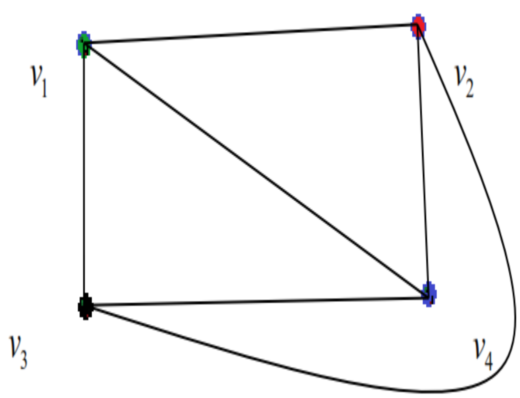
Definition 2.4. Regular fuzzy graph:

A fuzzy graph G is said to be regular if all its vertices have the same degree.In a fuzzy graph,if the degree of each vertex is 'k', that is $d(v)=\sum \mu(u,v)=k.$ Then the graph is called k-regular fuzzy graph.

Definition 2.5. Polyhedral fuzzy graph:

A simple connected planar fuzzy graph is called a polyhedral fuzzy graph if the degree of each vertex is greater than or equal to three. (i.e) $d(v)\geq 3$.
(i) $3|V|\leq 2|E|$ (ii) $3|R|\leq 2|E|$

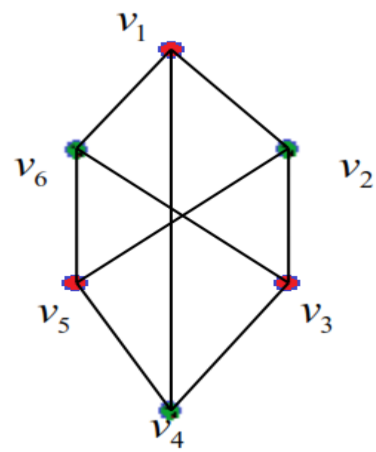
Example 2.5.1:



$d(V_1)=3$ $d(V_2)=3$ $d(V_3)=3$ $d(V_4)=3$

The degree of all vertices are equal to 3 and no edges are not intersect with other.Therefore the v_6 planar fuzzy graph is polyhedral fuzzy graph.

Example 2.5.2:

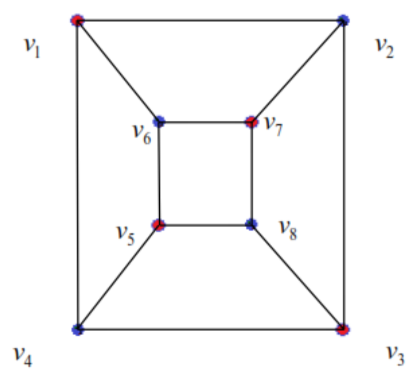


The degree of all vertices are same but the edges are intersect.So the graph is non-planar.The above example is not a polyhedral graph.

Definition 2.6. Polyhedral regular fuzzy graph:

A fuzzy graph is said to be polyhedral regular fuzzy graph if the degree of each vertex is ≥ 3 and each vertex has same degree that is $d(v)=\sum \mu(u,v)=k.$ Then no two edges are intersect other then the graph is called polyhedral regular fuzzy graph.

Example:2.6



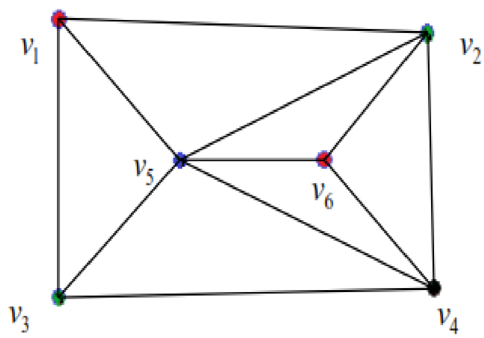
Here
$v_1=0.2, v_2=0.4, v_3=0.5, v_4=0.3, v_5=0.2, v_6=0.4, v_7=0.5,$
$v_8=0.5$
and
$\mu(v_1,v_2)=0.1, \mu(v_2,v_3)=0.1, \mu(v_3,v_4)=0.1, \mu(v_4,v_1)=0.1, \mu(v_4,v_5)=0.1,$
$\mu(v_5,v_6)=0.1, \mu(v_6,v_1)=0.1, \mu(v_6,v_7)=0.1, \mu(v_7,v_2)=0.1, \mu(v_7,v_8)=0.1,$
$\mu(v_8,v_3)=0.1, \mu(v_8,v_5)=0.1.$
The above graph is polyhedral regular fuzzy graph.if the degree of each vertex is 0.3 that is $d(v_1)=d(v_2)=d(v_3)=d(v_4)=d(v_5)=d(v_6)=d(v_7)=d(v_8)=0.3$

and no edges are intersect each other.So the graph is example of polyhedral regular fuzzy graph.

Definition 2.7.Polyhedral irregular fuzzy graph:

A fuzzy graph is said to be polyhedral irregular fuzzy graph if the degree of each vertex is ≥ 3 and each vertex has not same degree that is $d(v) \neq \sum \mu(u,v) \neq k$. Then no edges are intersect to other then the graph is called polyhedral irregular fuzzy graph.

Example: 2.7



Here $v_1 = 0.4, v_2 = 0.5, v_3 = 0.3, v_4 = 0.7, v_5 = 0.6, v_6 = 0.4.$
and

$\mu(v_1,v_2) = 0.3, \mu(v_2,v_4) = 0.1, \mu(v_4,v_3) = 0.1,$
$\mu(v_3,v_1) = 0.1, \mu(v_3,v_5) = 0.1, \mu(v_5,v_1) = 0.2,$
$\mu(v_5,v_2) = 0.3, \mu(v_5,v_6) = 0.1, \mu(v_5,v_4) = 0.3,$
$\mu(v_6,v_2) = 0.2, \mu(v_6,v_4) = 0.2.$

If the above graph,
$\deg(V_2) = 0.9$, $\deg(V_3) = 0.3$, $\deg(V_4) = 0.7$, $\deg(V_5) = 1.0$, $\deg(V_6) = 0.5$.

The degree of each vertex is greater than or equal three but degree of each vertex is not same.

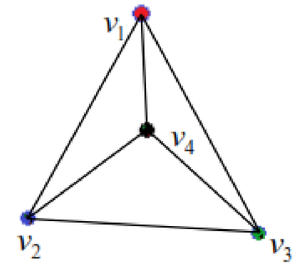
Therefore, the above graph is example an polyhedral irregular fuzzy graph.

III. Coloring of polyhedral regular and irregular fuzzy graph

Definition 3.1:Coloring of regular fuzzy graphs:

Let $G = (\sigma, \mu)$ be a fuzzy graph if $d(V) = K \forall v \in V$. That is if each vertex has same degree greaterthan or equal to 3 then G is said to be a polyhedral regular fuzzy graph. A Coloring of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring).Then the fuzzy graph G is said to be a coloring of polyhedral regular fuzzy graph.

Example:3.1



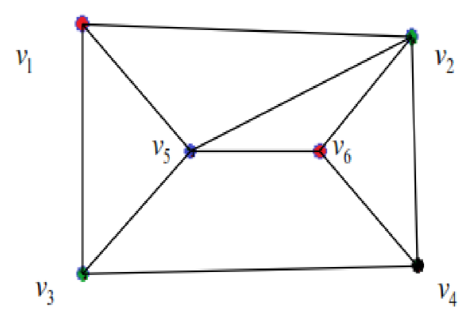
Here $v_1 = v_2 = v_3 = v_4 = 0.2.$
and
$\mu(v_1,v_2) = \mu(v_2,v_4) = \mu(v_4,v_3) = \mu(v_3,v_1) = \mu(v_3,v_4) = \mu(v_4,v_1) = 0.1.$

The above graph is polyhedral regular fuzzy graph.if the degree of each vertex is 0.3 that is $\deg(V_1) = 0.3$, $\deg(V_2) = 0.3$, $\deg(V_3) = 0.3$, $\deg(V_4) = 0.3$. Each vertex is adjacent so each vertex has the different color. Therefore the graph is 4-coloring of polyhedral regular fuzzy graph.

Definition 3.2:Coloring of irregular fuzzy graphs:

Let $G = (\sigma, \mu)$ be a fuzzy graph if $d(V) = K \forall v \in V$. That is if each vertex has not same degree greaterthan or equal to 3 then G is said to be a polyhedral regular fuzzy graph. A Coloring of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring).Then the fuzzy graph G is said to be a coloring of polyhedral irregular fuzzy graph.

Example:3.2



Here
$v_1 = 0.4, v_2 = 0.5, v_3 = 0.3, v_4 = 0.7, v_5 = 0.6, v_6 = 0.4.$

and
$\mu(v_1,v_2) = 0.3, \mu(v_2,v_4) = 0.1, \mu(v_4,v_3) = 0.1,$
$\mu(v_3,v_1) = 0.1,$
$\mu(v_3,v_5) = 0.1, \mu(v_5,v_1) = 0.2, \mu(v_5,v_2) = 0.3,$
$\mu(v_5,v_6) = 0.1, \mu(v_6,v_2) = 0.2, \mu(v_6,v_4) = 0.2.$

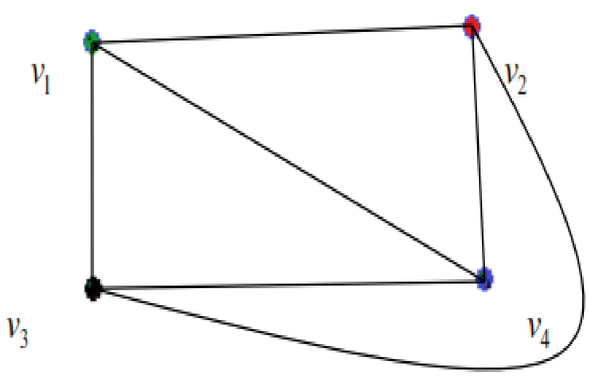
If the above graph $\deg(V_1)=0.6$, $\deg(V_2)=1.0$, $\deg(V_3)=0.3$, $\deg(V_4)=0.5$, $\deg(V_5)=1.0$, $\deg(V_6)=0.5$.

No.of.vertices	Adjacent	Not adjacent
V_1	V_2, V_3, V_5	V_4, V_6
V_2	V_1, V_4, V_5, V_6	V_3
V_3	V_1, V_5, V_4	V_2, V_6
V_4	V_2, V_3, V_6	V_1, V_5
V_5	V_1, V_2, V_3, V_6	V_4
V_6	V_2, V_4, V_5	V_1, V_3

No two adjacent vertices have the same color.
$\therefore$ The fuzzy graph has 3 coloring of polyhedral irregular fuzzy graph.

Theorem 3.1:
If the fuzzy graph $G(A,B)$ is polyhedral regular fuzzy graph if each vertex has greaterthan or equal to 2 colorable.
Proof:
Let $G(A,B)$ be a fuzzy graph.If $d(V)\geq K \forall v \in V$ .if each vertex has same degree K then the graph G is polyhedral regular fuzzy graph.
To prove:

If each vertex has greaterthan or equal to 2 color. By definition of coloring no two adjacent vertices have the same color.We apply the definition of polyhedral regular fuzzy graph.



V_1, V_2, V_3, V_4 are vertices.

No.of.vertices	Adjacent	Not adjacent
V_1	V_2, V_3, V_4	–
V_2	V_1, V_3, V_4	–
V_3	V_1, V_4, V_2	–
V_4	V_1, V_2, V_3	–

Every vertex is adjacent to other vertices.So we put different color in the fuzzy graph. V_1 has green color, V_2 has red color, V_3 has black color and V_4 has blue color.We using 4 color. Take any polyhedral graphs.If each vertex has greater than equal to 2 color.
Conversely,

Take each vertex has greater than equal to 2 color.
To prove:
The graph is polyhedral regular graph.We know that every planar graph contains greater than equal to 2 colorable.By our definition "A simple connected planar fuzzy graph is called a polyhedral fuzzy graph if the degree of each vertex is greater than equal to 2 .That is $\deg(V)\geq 3$ no two edges are intersect.

Remark:
A polyhedral regular graph G is K -region colorable iff its dual G is K -vertex colorable.

Theorem 3.2:
Let $G(A,B)$ be a fuzzy graph then a polyhedral fuzzy graph cannot have exactly 7 edges and no polyhedral graph has 30 edges and 11 regions.
Proof:
(i)The degree of every vertex is atleast 3.

We have $3V \leq 2e$ (or) $V \leq \frac{2e}{3}$, and as before $f \leq \frac{2e}{3}$. So if there 7 edges $V \leq \frac{14}{3} = 4\frac{2}{3}$ and $f \leq 4\frac{2}{3}$.

Since f and V are integers,this means $V \leq 4$ and $f \leq 4$.

Hence, $V - e + f \leq 4 - 7 + 4 = 1$ .Which $\Rightarrow\Leftarrow$ Euler's formula.
(ii)If there were such a graph,then by Euler's formula, $V = e - f + 2 = 30 - 11 + 2 = 21$

Then $3V = 63 > 60 = 2e$ which $\Rightarrow\Leftarrow$ the result $3V \leq 2e$ .So there is no such polyhedral fuzzy graph.

V. Conclusion

In this paper we proposed to many concepts of regular and irregular polyhedral fuzzy graphs and coloring of regular and irregular polyhedral fuzzy graphs and determined many interesting theorems and results.The conception of regular and irregular polyhedral fuzzy graphs and coloring of regular and irregular polyhedral fuzzy graphs can be used in different areas.

Our next plan is to extend our research of fuzzification to the coloring of regular and irregular planar fuzzy graphs and we examine many results.

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