

# Modeling of Probe-Drogue Docking Success Probability for UAV Autonomous Refuelling

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**Abstract**— Docking process of UAV Autonomous Refueling is a critical issue during the docking phase of autonomous aerial refueling (AAR), and the successful docking between the probe and drogue need higher probability for an aerial refueling system. To cope with this issue, a novel and effective model based on the theory of stochastic process crossing target area is proposed. In order to ensure its accurate and easy application, according to prior information and assumptions for the movements of the probe related to the drogue, the probe-drogue docking success probability is converted to the probability of the probe located in the circle area of drogue. The temporal and spatial characteristics of the pointing error have been considered which makes the model of the docking success probability more accord with the actual situation. simulations were conducted to demonstrate the effectiveness of the proposed method. This model provides theoretical support for the design and verification of AAR's control system.

**Keywords**—Autonomous aerial refueling, UAV, stochastic process, docking success probability

## I. INTRODUCTION

Along with the development and utilization of autonomous aerial refuelling (AAR) technology, unmanned aerial vehicle (UAV) can significantly advances the payload and endurance capacity without the need to return to base. In recent years, UAVs are increasingly used in both military and civilian domain, accordingly, and probe-drogue autonomous aerial refuelling (PDAAR) has dramatically gained a great deal of attention due to its flexibility, efficiency and economy [1-5].

In PDAAR, one critical issue is how to accurately and safely dock the probe and drogue with the performance of the PDAAR controller. In order to design a high-performance controller, the kinematics of the oil receiver and the hose-drogue need to be model. First, as for the refuelling hose is a flexible body, when it is affected by the atmospheric turbulence, the space position of the drogue relative to the tanker is not fixed, and there is a phenomenon of float pendulum [6], which seriously affects the refuelling docking success rate. To deal with this problem, many research institutes have made efforts to the dynamic model of hose-drogue bushings [7-10] and control. The model and simulation of hose-drogue shaking phenomenon during the PDARS docking stage were studied in [7-8]. In [9], the dynamic behaviour of the hose-drogue whip phenomenon during the PDARS docking stage was modelled, and the influencing factors of the whip phenomenon were analysed. In addition, the wake turbulence of the tanker and

atmospheric turbulence can create uncertainty in the movement of the receiver. Many studies use a typical small-disturbance linear model, such as [10,11]. For air refuelling, the accuracy of the model is more demanding and it needs to reflect its dynamic characteristics under turbulence disturbances. Therefore, the literature [12] has made some modifications to the model of the receiver. The literature [13] gives the fly model of winged drone docking in the air. Moreover, the designer of the PDAAR controller also need to obtain the relative position and attitude between the probe and drogue during docking phase using all kinds of sensors and methods. Valasek et al. [14] developed a vision-based drogue pose estimation system called VisNav. In Refs. [15-17], the authors used colourful information or visible markers to calculate the relative pose information. A drogue pose estimation method based on infrared vision sensor is introduced with the general goal of yielding an accurate and reliable drogue state estimate[18].

As discussed above, the previous works mainly focused on modelling the movement of the drogue and designing controller of the receiver, few kinds of literature analysed the probe-drogue docking success probability. The existing literature[15] proposed a mathematical model based on the actuation error of the docking control system for the docking success probability, but it cannot accurately depict the docking success probability because only the spatial

characteristic of the actuation error is considered, while the error's time characteristic is ignored.

The rest of the paper is organized as follows. In Section II, we describe the relative movement between the probe and the drogue during the docking stage of AAR for UAV, and the theory of stochastic process crossing target area (SPCTA) is also introduced. In Section III, The random characteristics of pointing error which is the position deviation of the probe relative to the center of the drogue are illustrated. The mathematical model of the docking success probability is given, and Simulation results, comparisons, and analysis are shown in Section IV. Finally, we conclude this paper in Section V

**II. RELATED WORK**

The overview diagram of the AAR process is presented in [5]. Figure1 shows a two-dimensional cross section of the capture criteria (yellow area) and miss criteria (blue area). The radius of the drogue is  $R_c$ . During the docking phase, the probe should be kept in the yellow area as much as possible.

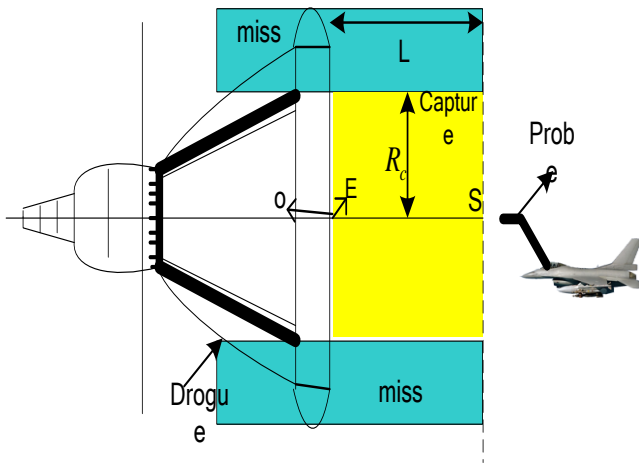


Figure1. The miss and capture criteria

Under the action of the flight control system, the receiver approaches towards the tanker at the speed of 0.3~0.5m/s. The AAR controller continues to measure and track to the actual drogue position in vertical and lateral directions, controlling the probe to point the drogue at positive longitudinal direction. During this process, the probe will swing randomly around the centre of the drogue plate ( $\Omega$ ), and the positional relationship of the pointing error ( $z(t)$ ) of the probe and the drogue is shown in Figure 3 for a docking process.

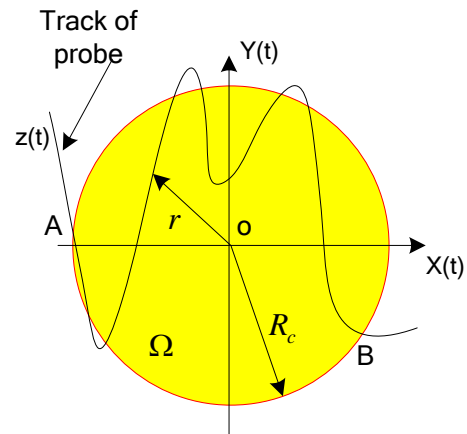


Figure2. Track of probe during the docking process

When the probe arrives at the drogue end plane where the S point lies in, as shown in the Figure 4, if the probe is in the yellow area, the docking is successful ( curve *a* in the Figure 4), otherwise it means failure (curve *b* in the Figure 4).

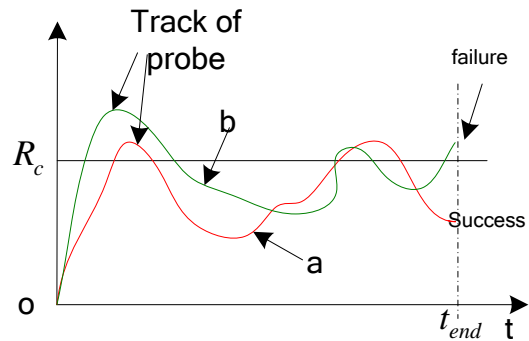


Figure3.Samples of the track of probe

**III. METHODOLOGY**

Assuming that the tracking system of the AAR adopts unbiased estimation and unbiased control strategy, and the pointing error  $z\{x, y\}$  is a stationary zero-mean Gaussian process, then the probability density function (PDF) of the pointing error is given by:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2\right]\right\} \quad (1)$$

Where  $x$  is the lateral displacement and  $y$  is the vertical displacement, and they are assumed to be statistically independent.  $\sigma_x$  and  $\sigma_y$  represent the standard deviation for the lateral and vertical displacements of the probe. The radial displacement  $r$  at the probe plane is expressed as

$$r = \sqrt{x^2 + y^2} \quad (2)$$

Further, if  $\sigma_x = \sigma_y = \sigma$  we can obtain the radial displacement  $r$  at the drogue plate by a Rayleigh distribution [11]:

$$f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad (3)$$

Then the probability of the pointing error locating in the circle region  $\Omega$  with radius  $R$  can be expressed as

$$\begin{aligned} \alpha_{in} &= P_{in} = \iint_{r \in \Omega} f(r) dr \\ &= \int_0^R \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) dr \quad (4) \\ &= 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right) \end{aligned}$$

Consequently, the outage probability of the pointing error is given by

$$\alpha_{out} = 1 - \alpha_{in} = \exp\left(-\frac{R^2}{2\sigma^2}\right) \quad (5)$$

Actually,  $\alpha_{in}$  only indicates the spatial probability of the pointing error to the region  $\Omega$ , as shown in Figure 3 and Figure 4, the pointing error appears periodically in and outside the  $\Omega$ , in other words, the pointing error randomly crosses bound threshold  $R$ . Considering the pointing error is a Gaussian process, therefore, the time characteristic of pointing error also needs to be discussed. Assuming the radial displacement is stationary Gaussian process, the ratio of expectation crossings of pointing error (crossing frequency) is given by [12]:

$$\lambda = \frac{\dot{\sigma}}{2\pi\sigma} \exp\left\{-\frac{1}{2}\left(\frac{R}{\sigma}\right)^2\right\} \quad (6)$$

The number of times the pointing error passes through the drogue plate can be seen as independent of each other. We define every period which the pointing error is in the capture criteria as in-area period, and the others are out- area periods. That means that the in- area period and the out- area period alternate periodically in a docking process, and they are also statistically independent. The two periods can approximately expressed by exponential distributions, respectively:

$$f_{in}(t) = \frac{\lambda}{\alpha_{in}} \exp\left\{-\frac{\lambda}{\alpha_{in}}t\right\} \quad (7)$$

And

$$f_{out}(t) = \frac{\lambda}{\alpha_{out}} \exp\left\{-\frac{\lambda}{\alpha_{out}}t\right\} \quad (8)$$

Time correspondences for the docking success probability are very critical in this work. Based on above analysis of the actual physical process in AAR, when the distance between the probe and drogue is  $L$  (about 6~8 meters) and their relative position is stable, the docking process begins. During the process, their relative speed  $v_r$  should be accurately controlled at 0.2~0.3m/s, and this speed exactly makes the probe hit the oil valve in the drogue without damage. So the time to complete a docking process can be calculated as shown in Eq. (9)

$$t_{docking} = L / v_r \quad (9)$$

As shown in Figure 6, during a docking process, the probe has two position state: in the  $\Omega$  or in the  $\bar{\Omega}$ , we define  $X_t$  as the status of the point error at  $t$  moment, and the state space of the point error is defined as  $S = \{0,1\}$ , then  $X_t$  can be viewed as a homogeneous Markov chain in the time domain, its transfer probability matrix is given as shown in Eq. (10).

$$P(t) = \begin{pmatrix} p_{00}(t) & p_{01}(t) \\ p_{10}(t) & p_{11}(t) \end{pmatrix} \quad (10)$$

Further, the transfer intensity matrix of  $X_t$  is represented by the definition and properties of continuous-time Markov chains

$$Q = \begin{pmatrix} -\lambda / \alpha_{in} & \lambda / \alpha_{in} \\ \lambda / \alpha_{out} & -\lambda / \alpha_{out} \end{pmatrix} \quad (11)$$

Considering that  $Q$  satisfies the post-differential equation group by

$$P'(t) = QP(t)$$

And then

$$\begin{cases} \lambda / \alpha_{in} p'_{00}(t) + \lambda / \alpha_{out} p'_{10}(t) = 0 \\ \lambda / \alpha_{in} p'_{01}(t) + \lambda / \alpha_{out} p'_{11}(t) = 0 \end{cases} \quad (12)$$

And then  $p_{00}(t)$  can be calculated using the constant variation method

$$p_{00}(t) = \alpha_{in} + \alpha_{out} \exp\left[-\left(\frac{\lambda}{\alpha_{in}} + \frac{\lambda}{\alpha_{out}}\right)t\right] \quad (13)$$

$p_{00}(t)$  represents the probability density function that the starting location and ending location of the probe are all in the  $\Omega$ . The position of probe is usually in the capture criteria at the beginning of the docking, and if the probe still in the drogue plate at  $t$  moment, that means the docking is successful. So, the docking success probability is just equal to  $p_{00}(t)$  on  $t \in [0, T]$ . Namely the docking success probability of UAVs-AAR is modeled in Eq. (10).

$$P_{docking}(t)|_{t=T} = p_{00}(T) = \alpha_{in} + \alpha_{out} \exp\left[-\left(\frac{\lambda}{\alpha_{in}} + \frac{\lambda}{\alpha_{out}}\right)T\right] \quad (14)$$

Where  $T$  is the time required for the docking, and its value is given by the in Eq. (9)

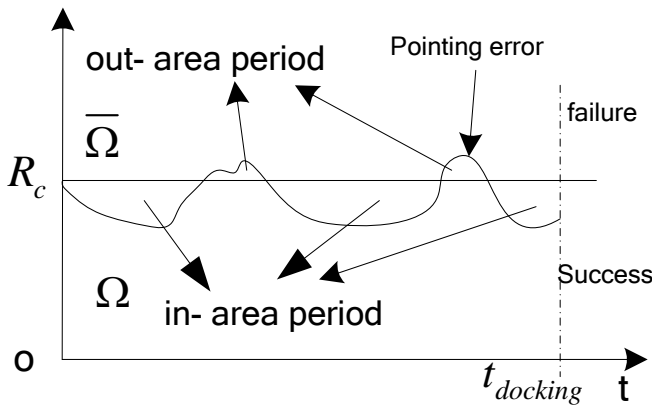


Figure4. In-area period and out-area period in a docking process

#### IV. RESULTS AND DISCUSSION

In this section we consider an autonomous aerial refuelling system of UAVs with PDR model that is subject to pointing error. Some parameters of the docking success probability model that have been derived in previous sections will be studied. Finally, an example of the calculation of the successful docking probability will be given.

##### A. crossing frequency parameter

Crossing frequency  $\lambda$  indicates the times of probe escaping from the drogue plate in unit time. The parameter  $\lambda$  represents the rapidity of the system. In order to ensure the safety of the docking process, we hope that the value of  $\lambda$  will be as small as possible. However, if the  $\lambda$  is too small, it means that the docking system will lose its Rapidity. Figure5 shows the relationships of the  $\lambda$  and its parameters ( $\dot{\sigma}, \sigma, R$ ).

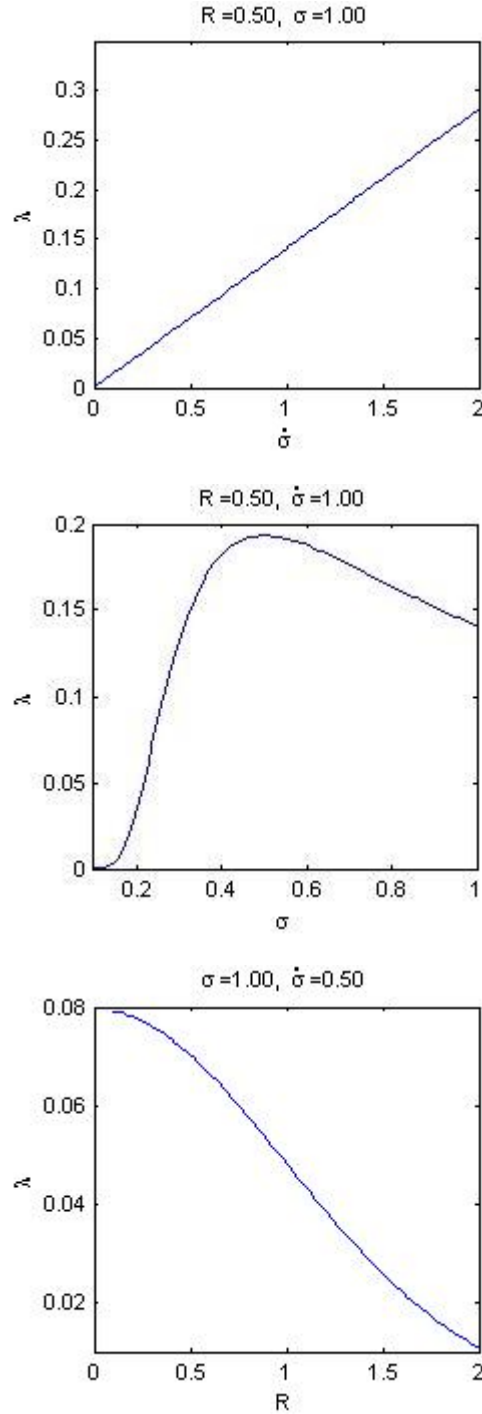


Figure5. The relationships of the  $\lambda$  and its parameters

##### B. Docking Success Probability

The docking success probability of UAVs-AAR is determined by the  $\alpha_{in}, \alpha_{out}, \lambda$  and  $T$ , and the algorithms of these parameters have been given in section III. There is an example which uses to display how to calculate the docking success probability as follow: assume that the probe

is approaching to the drogue at 2.0ft/s, the distance is 20ft between them, then the time of the docking is 10s, the result of the docking success probability is shown in figure6.

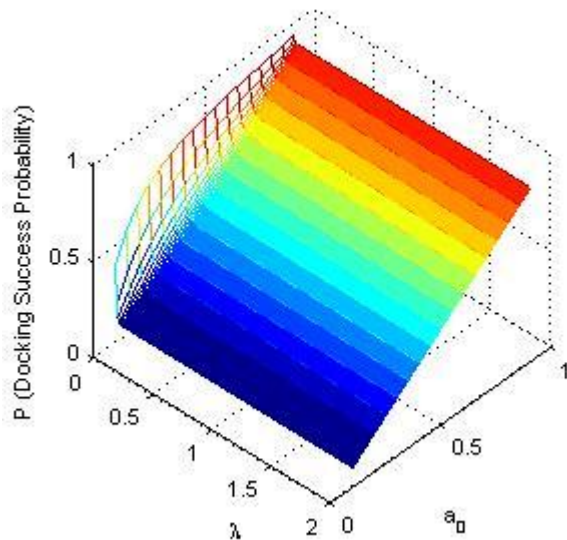


Figure6. A result of the docking success probability

## V. CONCLUSION and Future Scope

This paper has demonstrated the applicability of stochastic process crossing circle area to the docking success probability of UAV estimating, and highlighted some of their critical advantages for this application. We have shown the advantage of using the Spatial-temporal characteristics of the stochastic variables for coupling the UAV's docking process model. Logical assumptions for the distribution of the pointing error of the probe under atmospheric disturbance are acceptable, according to the prior information during the docking phase of AAR. We derived the exact expression for the docking success probability with three parameters which makes the design of the control system for ARR to be easier than before. In future, we are planning to use the actual flight data to verify the model.

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